ELECTROMAGNETIC WAVES GENERATED BY THE EMISSION OF GAMMA QUANTA IN AN INHOMOGENEOUS ATMOSPHERE

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1. It is well known that when a pulse of gamma quanta is emitted into the air, currents of Compton electrons are generated in the latter, and they in turn excite electromagnetic fields. If the gamma yield is isotropic and the surrounding medium is homogeneous, then the only nonzero component of the electromagnetic field is the radial electric field, which exists only in the current zone. The actual deviations from spherical symmetry in the current system result in the generation of other components of the electromagnetic field and in the radiation of electromagnetic waves. The influence of the various factors responsible for asymmetry of the currents has been investigated in several papers. For example, the influence of asymmetry of the gamma output has been studied in [1, 2], the influence of the underlying surface in [3, 4], and the influence of external fields in [5-7]. The emission of gamma quanta into an inhomogeneous atmosphere has been investigated in [8], in which are given the results of numerical calculations of the excited fields.

A direct comparison of the results of these studies shows that in the case where the effect evolves near the earth's surface the associated current asymmetry yields the maximum (in amplitude of the transverse fields) effects. With an increase in the height of the source h the influence of the underlying surface diminishes. This trend can be witnessed, for example, in the results of [4]. Thus, if expression (2.6) in that paper is used and the Compton electron currents are substituted for j as the upper bound of the radiated field (actually the radial currents are smaller due to the compensating contribution of conduction currents in them, hence the upper bound) we find that the amplitude of the radiated field decreases with increasing h approximately as exp $(-\rho_m/l_\gamma)$, where ρ_m is the minimum value of the lower limit in the integral (2.6) in [4], equal to h, and l_γ is the mean free path of the gamma quanta. On the other hand, as will be shown presently, the amplitude of the transverse fields, whose radiation is associated with the inhomogeneity of the atmosphere, depends weakly on the height of the source, and so, beginning with a certain height, this effect becomes the prevailing one. A comparison of these results with those of [3] affords an estimate of the threshold height $h_{th} \approx l_\gamma \ln 50\, Y$ (where Y is the activity of the source in the units used in [2, 8]). The results of [8] refer only to one concrete value of the source activity and its height, so that additional calculations are required in order to determine the dependence of the results on these parameters. An interesting possibility is the derivation of an analytical expression for the time dependence of the radiated field by approximate solution of the Maxwell equations. Such a possibility, obviously, would significantly facilitate the analysis of the results.

The present article is devoted to these two problems.

2. The statement of the problem of calculating the fields is essentially the same as that described in [8], except that the algorithm of [3, 6] is used for numerical integration of the Maxwell equations. Certain disparities in the results can be attributed to differences in the constants used. Those given in [8] (for example, the mobility and lifetime of secondary electrons, their number generated by one Compton electron, etc.) are taken below to be the same as in [8]. The values of certain constants are not given in [8]. For example, the space scale H of the density variation of the atmosphere and, accordingly, the density of the air at the source height is left out, as is the scale used to reduce the time to dimensionl form in the problem and, accordingly, the gamma-output duration 2Δ. Below, the inhomogeneity scale of the atmosphere H is taken equal to 6.6 km, and the distance r and time t are reduced to dimensionless form relative to the mean free path of the gamma quanta l_γ^* at zero height and to the ratio l_γ^*/c, respectively. In these units, the quantity Δ is chosen equal to 0.25.

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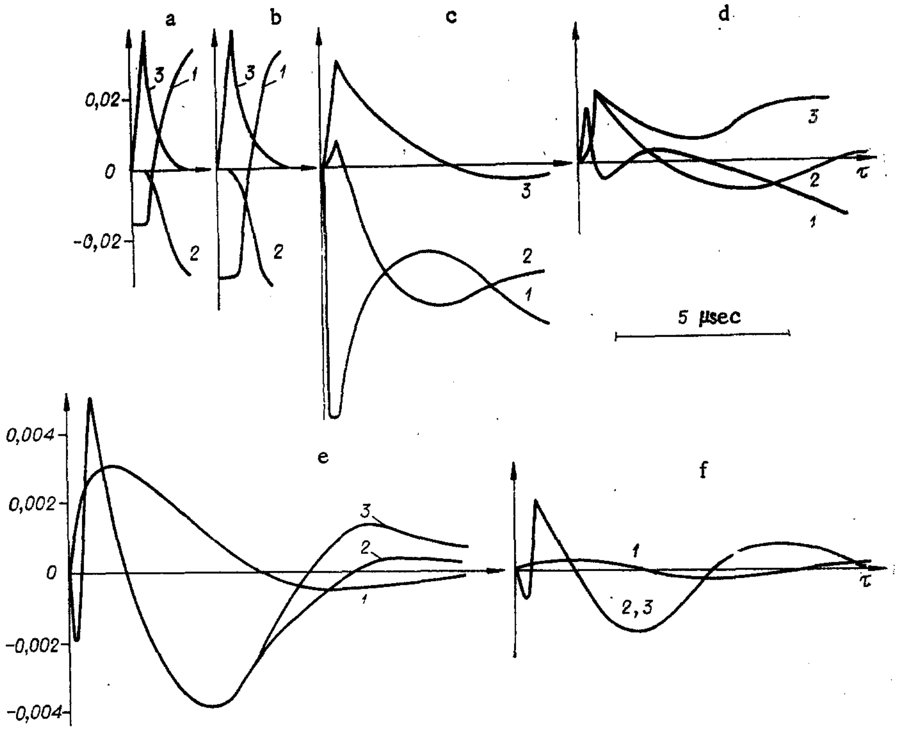

Fig. 1

The quantity ξ denotes the relative air density at the source height. The values of the fields are in cgs esu.

Figure 1 gives the time variations of the field components at various distances from the source ($Y = 1$, $\xi = 0.7$), complementing the results of [8]: 1) E_r; 2) E_ϑ; 3) B_φ; a) distance r = 0.1 km; b) 0.2 km; c) 0.5 km; d) 1 km; e) 2.5 km; f) 5 km. It follows from these results that in the current zone (to r ~ 0.5 km) the radial electric field E_r in a layer of thickness ~2Δ is near the leading edge of the gamma stream, where the currents and conductivity of the air are large, rapidly attain their limiting values (which differ at different distances), and remains approximately constant. Outside this layer the field varies due to induction effects. The transverse electric field E_ϑ at these distances in the conducting layer near the leading edge is considerably smaller than the radial field, while in the outer region, where induction effects are appreciable, it is commensurate with the radial field. Outside the current zone (at distances ≥1 km) induction effects prevail; here a radiation field is formed in which $B_\varphi \sim E_\vartheta$, but the value of the radial field is still commensurate with the transverse field. At distances r ~ 5 km the wave zone begins, where the radial electric field is much smaller than the transverse and essentially the radiation field has evolved.

Figure 2 illustrates the influence of variation of the source activity; these results are plotted for $Y = 10$, $\xi = 0.7$, and the nomenclature of the curves is the same as in Fig. 1. The amplitude of the radial field in the current layer is practically independent of Y, but, as should be expected, the size of the current zone increases with Y. Thus, ar r = 1 km the amplitude of E_r is considerably greater than the amplitudes of the fields E_ϑ and B_φ for $Y = 10$, whereas for $Y = 1$ the field amplitudes are commensurate at this distance. The evolution of the wave field takes place at greater distances for $Y = 10$ than for $Y = 1$. Thus, at r = 5 km the differences in the time variations of E_ϑ and B_φ are practically indiscernible if $Y = 1$, but for $Y = 10$ they are appreciable. The values of the fields E_ϑ and B_φ increase with Y, being roughly proportional to Y at small distances (less than r ~ 1 km).

With an increase in the source height the amplitude of the radial field in the current layer near the leading edge decreases (approximately as ξ^2), and the amplitudes of the induction fields vary only slightly. The dimensions of the current zones and the time scales of the variation of the induction fields increase approximately as ξ^{-1}. These properties of the generated electric field are illustrated in Fig. 3 ($Y = 10$, $\xi = 0.5$, same nomenclature as in Fig. 1).

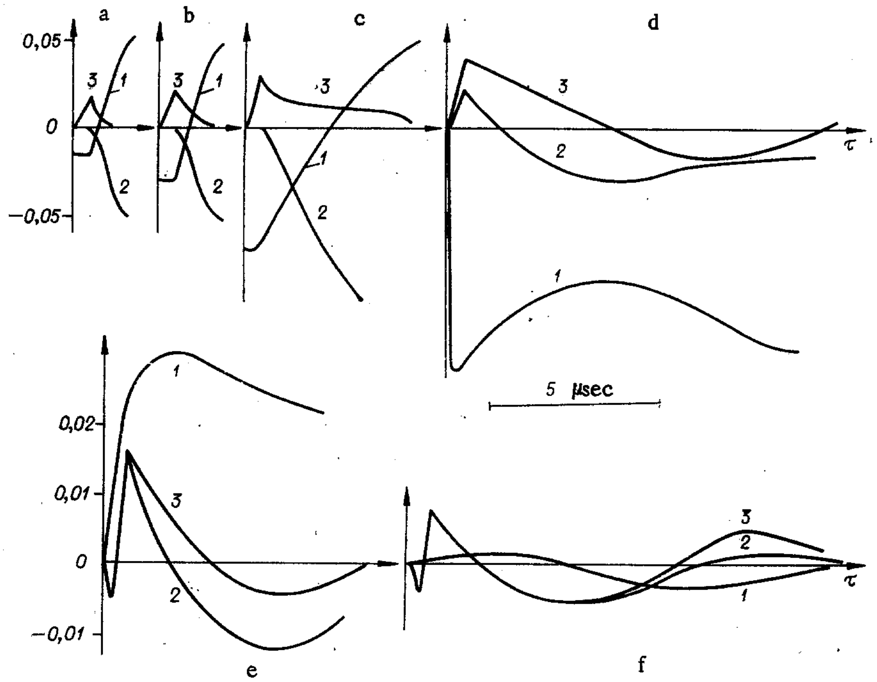

Fig. 2

The amplitude–time curves of the fields radiated at a distance of 100 km are shown in Fig. 4: 1) Y = 1, ξ = 0.7; 2) Y = 10, ξ = 0.7; 3) Y = 10, ξ = 0.5. It follows from these data that increasing the source height (while keeping the same total quantum yield) does not alter the field in the wave signal, but does increase (as ξ^{-1}) the time scales of the field variations. An increase in the total quantum yield causes both the field and the characteristic time scales of the signal to increase.

3. We now discuss briefly the behavior of the fields in the current zone near the leading edge of the gamma stream. In this region the spatial variations of the fields are characterized by two scales, one along the leading edge and one perpendicular to it. The longitudinal scale L coincides with the actual extent of the current zone. In the direction perpendicular to the leading edge the field variations are determined by the conductivity σ and the "thickness" of the current layer, i.e., by the product of the gamma emission time and the speed of light c. It can be assumed in regard to the motion of the leading edge with the speed of light that near it all quantities depend only on the "local" time $\tau = t - r/c$, so that only the derivatives with respect to τ remain in the equations. For example, the curl of the mangetic field **B** can be written in the approximate form (rot = curl):

$$\operatorname{rot}\mathbf{B} = \left[\frac{\partial}{\partial \mathbf{r}}\times\mathbf{B}\right] \approx \frac{\partial}{\partial r}[\mathbf{n}\times\mathbf{B}] \approx -\frac{1}{c}\frac{\partial}{\partial\tau}[\mathbf{n}\times\mathbf{B}] \qquad (3.1)$$

(**n** is the unit vector normal to the leading edge). We use this fact to deduce the following relation from the Maxwell equations:

$$\mathbf{n}\frac{\partial}{\partial\tau}(\mathbf{n},\mathbf{E}) + 4\pi(\sigma\mathbf{E} + \mathbf{j}_c) = 0, \qquad (3.2)$$

which after scalar multiplication by **n** yields the equation for the radial electric field

$$\partial E_r/\partial\tau + 4\pi(\sigma E_r + j_c) = 0. \qquad (3.3)$$

Equation (3.3) is strictly fulfilled only for fields excited by spherically symmetric currents in a homogeneous medium, but it can be used for an approximate determination of E_r when σ and j_c depend, e.g., on the polar angle ϑ, as in the investigated case of fields in an inhomogeneous atmosphere. Assuming, as in [8], that the effects of inhomogeneity are small, i.e., writing

$$\mathbf{j}_c = j_0(r,\ t) + j_1(r,\ t)\cos\vartheta,\ \ \sigma = \sigma_0(r,\ t) + \sigma_1(r,\ t)\cos\vartheta,\ \ E_r = E_0(r,\ t) + E_1(r,\ t)\cos\vartheta,$$

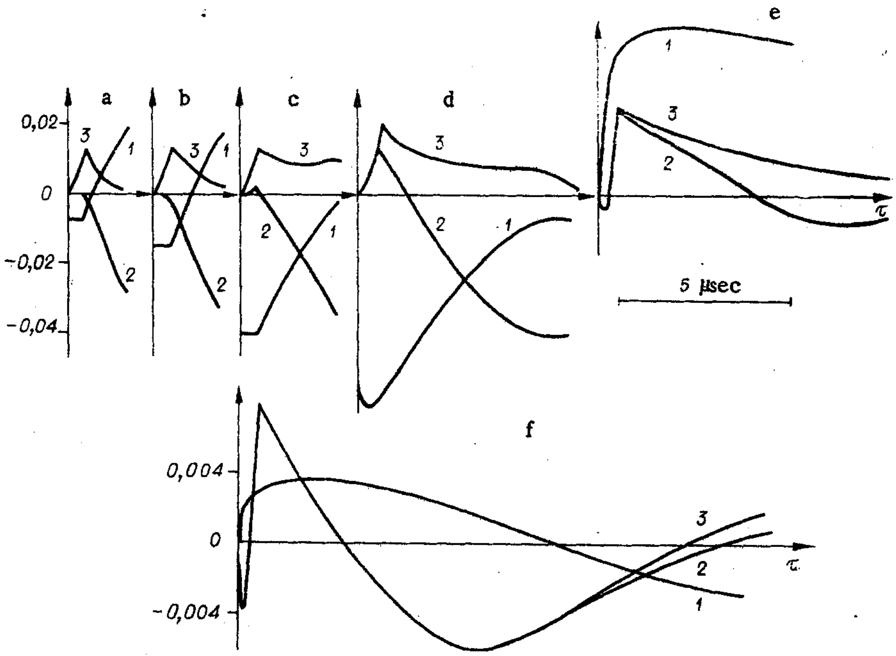

Fig. 3

where $j_1 << j_0$, $\sigma_1 << \sigma_0$, $E_1 << E_0$, for the determination of the fields E_0 and E_1 we have the system of equations

$$\partial E_0/\partial\tau + 4\pi(\sigma_0 E_0 + j_0) = 0,\ \ \partial E_1/\partial\tau + 4\pi\sigma_0 E_1 + 4\pi(\sigma_1 E_0 + j_1) = 0, \tag{3.4}$$

which are solved successively to obtain

$$\begin{aligned} E_0 &= -4\pi\int_0^\tau j_0(\tau')\exp\left\{-4\pi\int_{\tau'}^{\tau}\sigma_0(\tau'')\,d\tau''\right\}d\tau', \\ E_1 &= -4\pi\int_0^\tau (\sigma_1 E_0 + j_1)\exp\left\{-4\pi\int_{\tau'}^{\tau}\sigma_0(\tau'')\,d\tau''\right\}d\tau'. \end{aligned} \tag{3.5}$$

At small distances, where $\sigma_0\tau_0 >> 1$, we can estimate the integrals by the Laplace method:

$$E_0 \approx -j_0(\tau)/\sigma_0(\tau),\quad E_1 \approx [\sigma_1(\tau)\,j_0(\tau) - \sigma_0(\tau)\,j_1(\tau)]/\sigma_0^2(\tau). \tag{3.6}$$

If (following [8]) we assume that the conductivity of the air is electronic and that the conduction electrons themselves, being generated in events of interaction between Compton electrons and air molecules, vanish, becoming attached to oxygen molecules in a time $\sim\gamma^{-1}$ (the capture rate $\gamma \approx 10^{-8}$ sec^{-1} at zero height and varies as ξ^2 with increasing height), then with error $\sim(\gamma\tau_0)^{-1}$ the time dependence of the functions j_0, j_1, σ_0, and σ_1 can be regarded as identical, implying constancy (in time) of the fields E_1 and E_0 in the current zone near the leading edge of the gamma stream. This result is consistent with numerical integration (see Figs. 1-3).

Within the context of the stated assumptions the value of the field E_0 near the source does not depend on the distance [E_0 $(r = 0) = E_* \approx 1$ cgs esu at zero height and varies as ξ^2 with increasing height], while the field E_1 varies with distance according to the law $E_1 = E_*(2r/H)$.

At large distances from the source the inequality $\sigma_0\tau_0 >> 1$ no longer holds, and the fields E_0 and E_1 cannot grow to their limiting values (3.6) during the active period of the gamma stream. In this region we obtain from (3.5)

$$\begin{gathered} E_0(r, \tau\to\infty) = E_*[1 - \exp(-\Sigma)], \\ E_1(r, \tau\to\infty) = E_*(2r/H)\{[1 - \exp(-\Sigma)] - [(r/4l_\gamma) + 1]\Sigma\exp(-\Sigma)\}, \\ \Sigma(r) \equiv 4\pi\int_0^\infty \sigma_0(\tau, r)\,d\tau. \end{gathered}$$

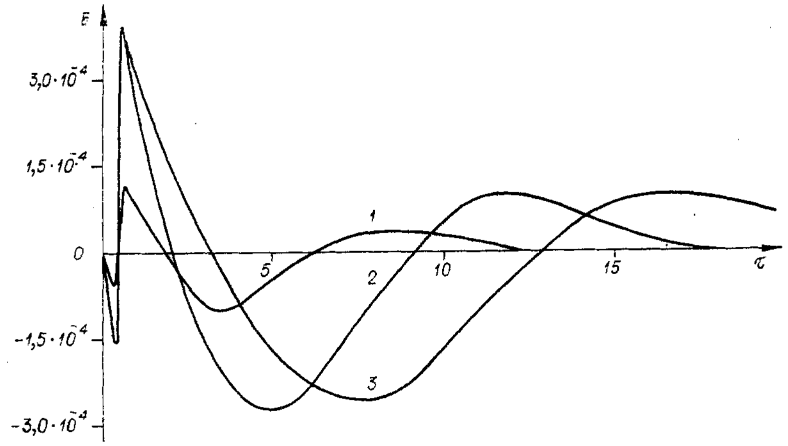

Fig. 4

If we attempt to determine E_ϑ from Eq. (3.2), then, taking the vector product of all terms with **n**, we obtain $[\mathbf{n} \times \mathbf{E}] = 0$. This result indicates that $E_\vartheta \ll E_r$ in the current zone. This conclusion is also consistent with the results of numerical integration.

4. We now turn our attention to the radiated field. Its determination does not present any difficulties if the radiating currents are known. These can be interpreted as the Compton electron currents, compensated by the radial induction currents, and the transverse conduction currents. We show that the latter play an insignificant role and can be neglected. We consider the ϑ component of one of the Maxwell equations:

$$\operatorname{rot}\mathbf{B} = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi}{c}(\sigma\mathbf{E} + \mathbf{j})$$

and the φ component of the other:

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}.$$

If we replace the curls of the fields **E** and **B** in the region near the leading edge by approximate expressions analogous to (3.1), then for E_ϑ and B_φ we obtain a homogeneous system of equations. Since $E_\vartheta = B_\varphi = 0$ at the leading edge of the current layer, the problem of determining E_ϑ and B_φ does not have any solutions other than the null solution. In the next-higher approximation we keep the term $(1/r)(\partial E_r/\partial\vartheta)$ in the expression for curl E. We obtain

$$\frac{\partial E_\vartheta}{\partial\tau} - \frac{\partial B_\varphi}{\partial\tau} + 4\pi\sigma E_\vartheta = 0, \quad \frac{\partial B_\varphi}{\partial\tau} - \frac{\partial E_\vartheta}{\partial\tau} + \frac{c}{r}E_r = 0,$$

whence we infer the relation

$$\frac{4\pi\sigma}{c}E_\vartheta + \frac{1}{r}E_r = 0.$$

In the current zone $4\pi\sigma \gg c/r$, and so $E_\vartheta \ll E_r$ (cf. Sec. 3), but it also follows from this result that the transverse conduction currents associated with E_ϑ are considerably smaller than the radial conduction currents associated with E_r. These currents are commensurate in the region where the conductivity is small. In this region, however, the actual currents are also small and can therefore be neglected. On the basis of this fact we take the radiating currents to be simply $j_r \equiv (j_1 + \sigma_1 E_0 + \sigma_0 E_1)\cos\vartheta$, where, as was explained in Sec. 3, in the region where they are large the field E_1 can be determined from the second equation (3.4), i.e., we can put

$$j_r = -\frac{1}{4\pi}\frac{\partial E_1(r,\tau)}{\partial\tau}\cos\vartheta \tag{4.1}$$

and specify $E_1(r, \tau)$ according to (3.5).

The vector potential A of the wave field is determined from the known radiating currents by means of the integral

$$A = \frac{1}{Rc}\int dV j_v\left(r, t - \frac{R}{c} + \frac{(\mathbf{r}, \mathbf{n})}{c}\right),$$

in which R is the distance to the detection point, $\mathbf{n}$ is the unit vector in the direction of that point, and j_V is the vertical component of the radiating currents. In a spherical coordinate system (r, ϑ, φ) with the z (= cos φ) axis directed vertically upward, $dV = r^2 dr \sin\vartheta\, d\vartheta\, d\varphi$, $j_V = j_r \cos\vartheta$, $(\mathbf{r}, \mathbf{n}) = r \sin\vartheta \cos\varphi$, if the direction of $\mathbf{n}$ is horizontal and the angle φ is measured from it. Knowing from (4.1) that the function j_r depends on r and $t - r/c$, we can write the vector potential in the form

$$A = \frac{2}{Rc} \int_0^\infty r^2 dr \int_{-1}^{1} x dx \int_0^\pi d\varphi j_r \left[r, x, \tau - \frac{r}{c}\left(1 - \sqrt{1 - x^2} \cos\varphi\right)\right],$$

where $x = \cos\vartheta$; $\tau = t - Rc$. If we replace the variable φ by $\tau' = \tau - r(1 - \sqrt{1 - r^2} \cos\varphi)/c$, then after appropriate manipulations we obtain

$$A = \frac{2\pi}{R} \int_0^\tau d\tau' c(\tau - \tau') \int_{c(\tau-\tau')/2}^{\infty} dr \left(1 - \frac{c(\tau - \tau')}{r}\right) j(r, \tau'), \tag{4.2}$$

$$j(r, \tau') \equiv j_r(x, r, \tau')/x.$$

The subsequent analysis of the integral (4.2) is exactly analogous to that in [4]. In particular, for $\tau < \tau_0$ the function $A(\tau)$ is determined by the time dependence of the radiating currents, and for $\tau > \tau_0$ it is determined by the spatial dependence of the time-integrated characteristics of the radiating currents, i.e., if the singular leading-edge properties of the signal can be disregarded, then in place of (4.2) we can calculate the potential according to the expression

$$A = \frac{2\pi}{R} c\tau \int_{c\tau/2}^{\infty} dr (1 - c\tau/2r) \int_0^\infty j(r, \tau') d\tau', \tag{4.3}$$

which with regard for (4.1) can be written in the form

$$A = -\frac{c\tau}{2R} \int_{c\tau/2}^{\infty} dr (1 - c\tau/2r) E_1(r, \tau' \to \infty),$$

where $E_1(r, \tau')$ is given by expression (3.6). Differentiating with respect to τ and substituting the function E_1, we obtain an equation for the time dependence of the field in the wave zone:

$$E = E_* \frac{1}{RH} \int_{c\tau/2}^{\infty} dr (r - c\tau) \{[1 - \exp(-\Sigma)] - [(r/4l_\gamma) - 1] \Sigma \exp(-\Sigma)\}. \tag{4.4}$$

Within the context of the above-stated (see Sec. 3) assumptions regarding the nature and time dependence of the conductivity of the air the quantity Σ is specified by the expression

$$\Sigma = \frac{e\mu\nu N}{\gamma l_\gamma^3} \frac{\exp(-s)}{s^2}, \quad s \equiv r/l_\gamma, \tag{4.5}$$

in which N in the total yield of gamma quanta and the rest of the notation is the same as in [8]. It is essential to note that the coefficients in front of the factor $s^{-2} \exp(-s)$, which describes the spatial dependence of Σ, does not depend on the height. If the variable r is made dimensionless by reference to l_γ, then the factor preceding the integral with respect to s will have the form $E_* l_\gamma^2/RH$, i.e., will also be independent of ξ. Consequently, the field E as a function of $T = c\tau/l_\gamma$ does not depend on the height, while as a function of τ it "stretches" with increasing height as ξ^{-1}, without changing its actual value. This result is fully compatible with Sec. 3 (see Fig. 4). The integral (4.4) with the function Σ given by (4.5) has been computed numerically for various values of Y. The results are given in Fig. 5 (field in cgs esu at a distance of 100 km). Inasmuch as the transformation of the field–time curve with increasing height is reducible to "stretching" of the scale in proportion to ξ^{-1}, the curves in Fig. 5 refer to a single value of the parameter $\xi = 0.5$; the curves are numbered as follows: 1) Y = 1; 2) 10; 3) 100. Curve 2 represents the time dependence of the radiated field for the same case as curve 3 in Fig. 4. A comparison of these curves shows that, exclusive of the singular features of the behavior of the field at the leading edge (for $\tau < 0.5$ μsec), the approximate expression (4.4) fully satisfactorily describes the radiated field. The discrepancies between the results for small values of τ are natural insofar as the transition

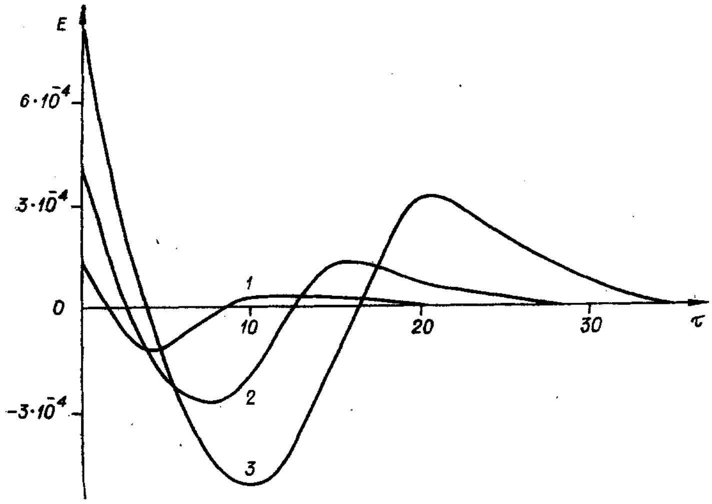

Fig. 5

from (4.2) to (4.3) is equivalent to neglecting the leading-edge singularities of the behavior of the radiated field.

The observed agreement of the results implies that the above-described approximate method is applicable to calculations of the configuration and amplitudes of the wave fields and the analysis of their variations with the parameters of the problem.

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